MathMind – Variations on MasterMind

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Mastermind type of games have been analyzed and solutions have been offered for optimal moves (Koyama and Lai [1]). Here we propose certain variations of this game that will be of interest to mathematicians/engineers/scientists.

The first variation concerns the way the response is made in Mastermind type of games. We generally assume numbers are used in the code word (if colors are used, they will be mapped to integers based on alphabetization or their position in the rainbow, etc.) and instead of the usual response 1 Right 3 Wrong, in this variation the response will be 1 Right 2 Higher 1 Lower indicating how the numbers in the wrong position are related. The numbers in the code word are the basis for the comparison and Higher (Lower) indicates the guessed number is higher (lower) than the corresponding number in the code word. Since the response contains more information, it will be interesting to see whether this type of game has lesser number of average and/or maximum guesses. Responses could also be of Mastermind variety with only two kinds of constituents like High or Low (the rest being equal) but pertain to all numbers in the response. Thus a response 2H3L for a six peg (or six number) game will indicate the guess has 2 Higher, 3 Lower and the rest Equal (in this case 1) numbers. This scheme has the elegance of a short response as well as non-ambiguity.

The second variation, personally more interesting, is that the type of response will be a single integer indicating the distance between the guess and the code word. Assuming an Euclidean distance type (let us not do square roots so as to keep distances as integers), the response for (1, 2, 3, 4) for a code word (2, 3, 1, 4) would be (1 – 2) ^ 2 + (2 – 3) ^ 2 + (3 – 1) ^ 2 + (4 – 4) ^ 2 = 6. It was proven that a maximum of 5 guesses are needed to solve the permutation type of mastermind games with 4 symbols [2]. A simple computational exercise on a PC will show that a maximum of two guesses are needed for this variation of the game. Here again of interest is the average/maximum number of guesses for the usual mastermind game with 6 numbers and 4 positions. Other distance criteria are equally possible (like absolute differences) but the squared distances have a mathematical appeal.

In the spirit of recreational (as opposed to boring) mathematics, I am leaving the explorations to younger (in spirit and age) generation of mathematicians and computer scientists. MathMind type of games (a name used to denote the above variations, especially the second one, of Mastermind) will become more interesting when the numbers are large, for instance for 10 symbols there are about 3.6 million possibilities and for 20, about 2.4E18 possibilities in permutation type of games. Of course there is enough mathematical structure in these games (like group theory, invariances, etc.) to make Galois happy. Also they will be ideal for games with internet access. Two players

in remote corners of the world can try to guess a 20-digit sequence with each one having access to his own computational resources and mathematical skills.

Discussions with Alexandre Temporel led me to certain finer refinements on the first variation of the game.

This is an internet publication and references to this paper can be made as:
Reddi, S. S., “MathMind – Variations on MasterMind,” http://www.rspq.org/pubs.

References:
[1] Koyama and Lai, “An Optimal Mastermind Strategy,” JRM, 1993-94.
[2] Reddi, S. S., “A Game of Permutations,” JRM, 8/1, 1975.