A Microsoft Puzzle

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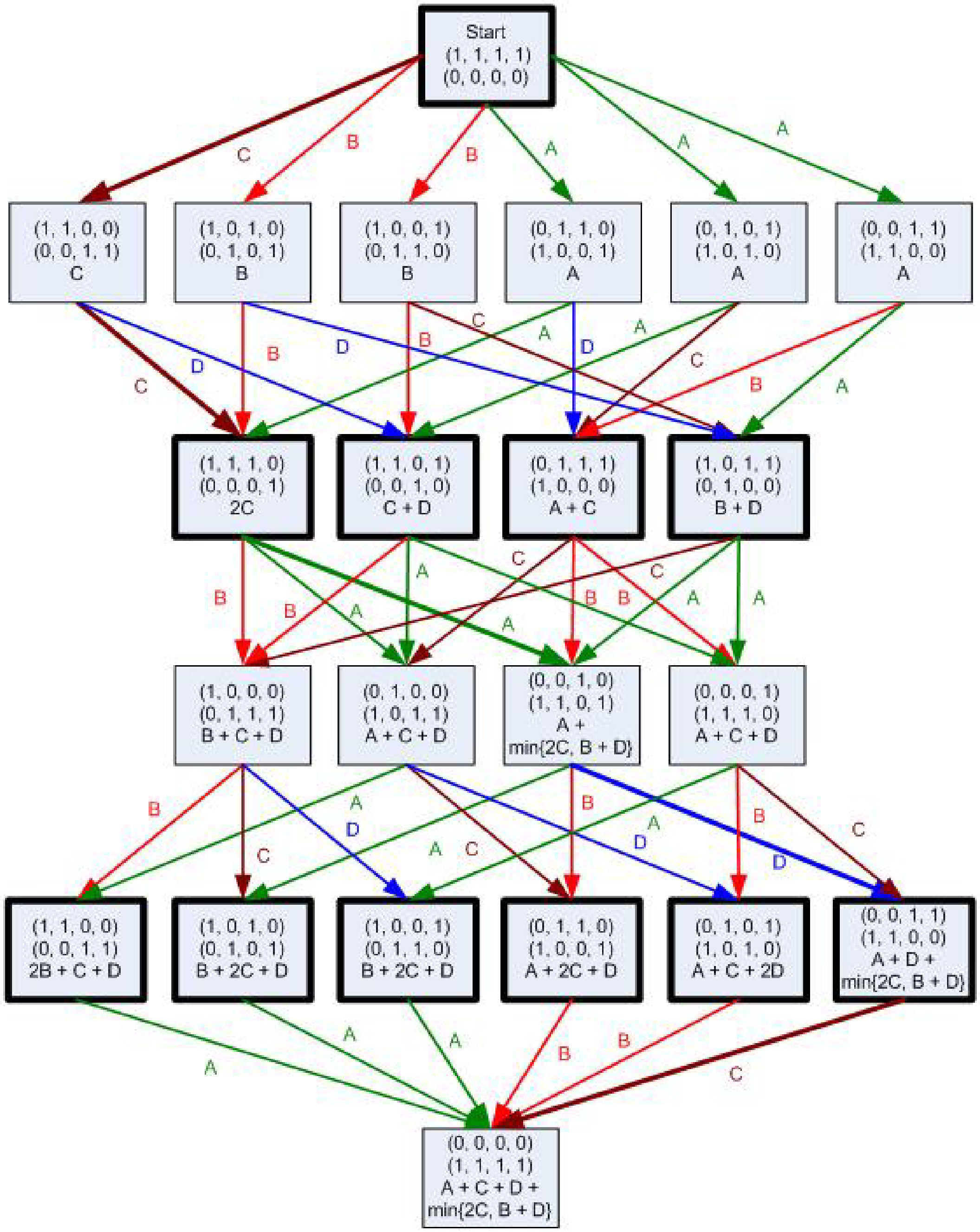
This puzzle, supposedly posed as a Microsoft question for their prospective employees, can be stated as follows:

There are four persons wishing to cross a bridge in the nighttime and they have one flash light. The bridge can take a maximum load of two persons and the persons can cross the bridge in 10, 5, 2, and 1 minute(s). When there are two persons crossing the bridge, the crossing time will be the slowest time of the two persons, e.g., if the two persons have crossing times of 10 and 2 minutes, the crossing time will be 10 minutes. For all four persons to cross the bridge, two persons have to cross the bridge at a time with the flash light and one person goes back with the flash light to fetch a new person. What is the strategy to have all the persons across the bridge ? The intuitive answer is 19 minutes with the fastest person as the runner with the flash light. However in this case it happens to be 17 minutes (it is not clear that when someone comes up with this answer that he is more eligible to work for Microsoft and it took me approximately two to three weeks for figuring out the exact solution – which makes me possibly ineligible to work for Microsoft).

The exact solution is $A + C + D + \min\{2C, B + D\}$ for persons with crossing times of $A \geq B \geq C \geq D$. The key thing to note is the relationship between the arithmetic mean of B and D, and C and this dictates whether the commonsense strategy prevails or otherwise. (Of course it is not clear whether the prospective Microsoft realizes this or not). The solution can be obtained by resorting to a state diagram and use a dynamic programming approach advocated by Bellman. (See the enclosed diagram). Each node represents a state with the persons on both sides of the bridge, and indicates the minimum time needed to reach that state. For instance node with the marking (1, 0, 1, 0), (0, 1, 0, 1) and B indicates the state with A and C at the starting side of the bridge, B and D on the other side, and that B is the time to reach that state.

There are interesting variations and extensions – what would be the conditions for five persons or more and are there inequalities that should hold for the commonsense case ? What happens when the crossing time for two persons with travel times t and s is a function $f(t, s)$ (e.g., $f(t, s) = (t + s)/2$) ?

Crossing Times: $A \geq B \geq C \geq D$



For crossing times A = 10, B = 5, C = 2 and D = 1, we get 17. For C = 3 or higher, we get the expected A + B + C + 2D