

Numerical Observations on Fermat Like Expressions

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We define expressions $z^n - x^n - y^n$ as Fermat Like Expressions (FLEs) and make certain numerical observations. x, y, z and n are taken to be positive integers and $z > y > x > 0$. Define defect $D(n)$ as the minimum of the absolute value of the expression $z^n - x^n - y^n$ for all such x, y and z , i.e.,

$$D(n) = \min\{|z^n - x^n - y^n|, n > 0, z > y > x > 0\} \quad (1)$$

and $T(n)$ as the triplet (x, y, z) that corresponds to the defect $D(n)$. Note the Fermat's Last Theorem (FLT) states that $D(n)$ is non-zero for $n > 2$.

Table 1 shows the defects (along with their signs and their corresponding triplets) for $x, y,$ and z in the range $1000 > z > x > y > 0$ and $n \leq 300$. Note the numbers are formidably long especially since we are showing the smallest numbers that are the defects. For an exponent of 300 and $x, y,$ and z around 1000, the numbers can be 900 digits long. We use Miracl package to compute multi-precision integers.

One may note that for exponents greater than 5, $(1, 2, 3)$ is the triplet with minimum defect and exponent 3 is the only one with defect 1, i.e., it is possible to find integers $x, y,$ and z such that $x^n + y^n + z^n = 1$. (Note it is known in the literature that such $x, y,$ and z exist for exponent 3 but it is not clear whether 3 is the only integer with this property). We generalize these observations into the following conjectures.

Conjecture 1: Triplet $(1, 2, 3)$ gives the defect for $n > 5$ and this defect is obviously always greater than 0.

Conjecture 2: Exponent 3 is the only positive integer with unit defect.

Proof of Conjecture 1 (without resorting to the proof of FLT) obviously establishes FLT as a corollary. Of course the proof may not be that simple as it took around 400 years to establish FLT.

Table 2 shows values for $z^n - x^n - y^n$ for $n = 10, 11$ in order to fathom the basis for Conjecture 1. We note that the value turns negative for $(20, 21, 22)$ for 10 and $(22, 23, 24)$ for 11, and remains negative for $(x, x+1, x+2), x \geq 20$ for $n = 10$ and $x \geq 22$ for $n = 11$. We will analyze when the defect turns negative for a given x, y, z and n , and give asymptotic values as $n \rightarrow \infty$.

Table 1. Defects for $n = 3 \dots 300$

n = 3 1 6 8 9 -1 9 10 12 -1 64 94 103 1 71 138 144 -1 73 144 150 1 135 138 172 -1 135 235 249 1 242 720 729 -1 244 729 738 -1 334 438 495 1 372 426 505 1 426 486 577 1 566 823 904 1 791 812 1010	n = 20 3485735824 1 2 3
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n = 5 -12 13 16 17	n = 22 31376865304 1 2 3
n = 6 664 1 2 3	n = 23 94134790218 1 2 3
n = 7 2058 1 2 3	n = 24 282412759264 1 2 3
n = 8 6304 1 2 3	n = 25 847255055010 1 2 3
n = 9 19170 1 2 3	n = 26 2541798719464 1 2 3
n = 10 58024 1 2 3	n = 27 7625463267258 1 2 3
n = 11 175098 1 2 3	n = 28 22876524019504 1 2 3
n = 12 527344 1 2 3	n = 29 68629840493970 1 2 3
n = 13 1586130 1 2 3	n = 30 205890058352824 1 2 3
n = 14 4766584 1 2 3	n = 31 617671248800298 1 2 3
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n = 17 129009090 1 2 3	n = 34 16677164519797384 1 2 3
n = 18 387158344 1 2 3	n = 35 50031510739261338 1 2 3
n = 19 1161737178 1 2 3	n = 36 150094566577522384 1 2 3
	n = 37 450283768452043890 1 2 3
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n = 223
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n = 224
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n = 225
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1540603010 1 2 3

n = 226
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1842307464 1 2 3

n = 227
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99967919258 1 2 3

n = 228
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n = 229
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n = 230
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n = 233
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n = 235
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2877425212753144 1 2 3

n = 239
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n = 240
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0370766351229884799424827749449111633457135229560
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n = 241
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25319936884278850 1 2 3

n = 242
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219172213238076104 1 2 3

n = 243
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143941444884707418 1 2 3

n = 244
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n = 246
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24370561608365306264 1 2 3

n = 247
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8474652943485878151882394980286095035807423741396
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n = 251
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n = 260
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n = 265
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75429616729822572465213712610 1 2 3

n = 266
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2462493031325405797511078841042813086277452676394
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n = 269
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5803207515728422685850791411594210763177588870656
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n = 270
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n = 271
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n = 272
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n = 273
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3180067514517270107317257158381199719404033018573
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66599020922195874067689870927476267410 1 2 3

n = 286
2862060763065543096585531442543079747463629716716
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n = 287
8586182289196629289756594327629239242390889150149
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2575854686758988786926978298288771772717266745044
8341805303337383631692601624966684765782528854888
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7727564060276966360780934894866315318151800235134
5075148233653129559293343099714736381357632179743
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n = 290
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n = 291
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n = 292
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n = 293
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9575522089992307206507807603028023178112741365196
153025984036198918679441826158038382473330 1 2 3

n = 294
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6874248061354203479207239504102476780122143869241
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n = 295
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0625927052775633072131512958695569993743074527089
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n = 296
1690018259982572543102790461507263160079798711423
9188414689575294448541412776886298928798250941999
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n = 297
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46878356983097184350806541704870530089096290 1 2 3

n = 298
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n = 299
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n = 300
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3

n = 301
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3

n = 302
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97830509357420820148967796415541544437237504504 1
2 3

n = 303
3696069934581886151765802739316384531094519781884
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6815527307480702244320914445677980136446103018 1
2 3

n = 304
1108820980374565845529740821794915359328355934565
2332321597319774996111391082294189547766121787309
953793309229510190327797793735140634058805488064 1
2 3

Table 2. Values for $z^n - x^n - y^n$ for $n = 10, 11$

>>>>> n = 10 <<<<<<<

58024 1 2 3	4108933741175 2 19 20	790217999 4 7 8
989526 1 3 4	10548814719376 2 19 21	2411994001 4 8 9
8717048 1 4 5	6439880977177 2 20 21	6512167023 4 9 10
50700550 1 5 6	16319922790400 2 20 22	15936376025 4 10 11
222009072 1 6 7	9880041812199 2 21 22	35978891047 4 11 12
791266574 1 7 8	24746630234424 2 21 23	75940079049 4 12 13
2413042576 1 8 9	14866588421201 2 22 23	151395114551 4 13 14
6513215598 1 9 10	36843458172928 2 22 24	287394687073 4 14 15
15937424600 1 10 11	21976869750703 2 23 24	810255924224 4 14 16
35979939622 1 11 12	53940920425952 2 23 25	522860188575 4 15 16
75941127624 1 12 13	31964050674225 2 24 25	1439342461248 4 15 17
151396163126 1 13 14	77763714686976 2 24 26	916481224097 4 16 17
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522861237150 1 15 16	64724036440249 2 26 27	4115071308776 4 17 19
1439343509823 1 15 17	155029671041024 2 26 28	2560597982601 4 18 19
916482272672 1 16 17	90305634599751 2 27 28	6669531724800 4 18 20
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4115072357351 1 17 19	294293233303552 2 28 30	6439879929625 4 20 21
2560599031176 1 18 19	523431520284353 2 28 31	16319921742848 4 20 22
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14866588422224 1 22 23	6513156550 3 9 10	77763713639424 4 24 26
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	4108933683150 3 19 20	35970173998 5 11 12
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50699527 2 5 6	9880041754174 3 21 22	810247207175 5 14 16
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1554473325151 2 17 18	294293233245527 3 28 30	14866578656600 5 22 23
4115072356328 2 17 19		36843448408327 5 22 24
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522578761902 7 15 16
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Consider the polynomial $F(x, n) = (x + 2)^n - (x + 1)^n - x^n$ which describes 3-tuples like (20, 21, 22). It is simple to show that the leading coefficient is -1 and the rest of them are positive, i.e., $F(x, n) = -x^n + k_1 x^{n-1} + k_2 x^{n-2} + \dots + k_n$ and $k_i > 0, i = 1 \dots n$. By Descartes' Sign Rule, $F(x, n)$ can have at most one positive and negative root.

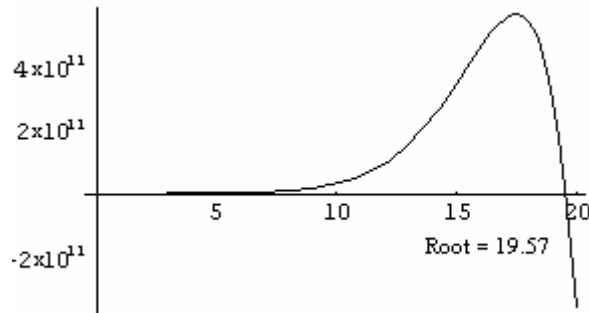


Figure 1.

Figure 1 shows a plot of $F(x, 10)$ (obtained by using Mathematica) and its roots can be computed as (again using Mathematica):

19.57, $-1.30195 \pm i 3.02143$, $-0.982686 \pm i 1.37549$, $1.00038 \pm i 0.726541$
 $1.0 \pm i 0.32492$, -1

The positive root r of $F(x, n)$ is in the vicinity of $2n$ for $n \leq 100000$ but very slowly increasing. This can be seen from the following table which shows the first triplet ($w, w+1, w+2$) whose value is negative :

Table 3. Relationship between $F(x, n)$ and its positive root

n	w	r / n
10	20	1.957
100	207	~2.065
1000	2077	~2.0765
10000	20780	~2.07795
100000	207808	~2.078075

In fact if $\alpha = r / n$, it can be shown that $\alpha \rightarrow 2.078177640.. = 1 / \ln \phi$ where ϕ is the golden ratio 1.618033..

We note that if $F(x, n)$ does indeed have an integral zero, we may proceed as follows. Assume n to be prime $p > 5$ and hence:

$$F(x, p) = 0 \Rightarrow (x + 2)^p - (x + 1)^p - x^p = 0 \pmod{p} \Rightarrow x^p = 1 \pmod{p}$$

Thus the root x has to be $x = 1 \pmod{p}$ or $x = kp + 1$ for some k . We also note:

$$F(x, p) = 0 \Rightarrow (x + 2)^p - (x + 1)^p - x^p = 0 \pmod{2} \Rightarrow (x + 1)^p = 0 \pmod{2}$$

and since $x = kp + 1$, we have $(kp + 1 + 1)^p = 0 \pmod{2} \Rightarrow (kp)^p = 0 \pmod{2}$ implying that k has to be even. Thus if a root r exists it should be of the form $r = kp + 1$ where k is even. Note $r/p = k + 1/p = \alpha \rightarrow 2.078177640..$ implying that k can be only 2.

From Table 3 we see for around $n = 10$, the root is at 19.57 (not at $2*10 + 1 = 21$ if it had an integral root), for around $n = 100$, the root is 206.5 (not at $2*100 + 1 = 201$ if it had an integral root), and for around $n = 10000$, the root is 20779.5 and not definitely 20001. (Technically we have to consider primes around these stated values of n but from the monotonously increasing nature of α – a fact we did not prove but can be proved – we can safely assume that for the primes around these values, the roots have similar properties to make the discussion simpler). Thus we have the assertion:

Assertion 1: $F(x, p) = (x + 2)^p - (x + 1)^p - x^p = 0$ has no solutions in integers for prime $p > 5$.

Note what all we are proving is $F(x, p) \neq 0$ for integer x and we are nowhere closer to proving that (1, 2, 3) gives the defect as the Conjecture 1 requires.

A study of polynomials $F(x, n) = (x + 2)^n - (x + 1)^n - x^n$ may show why Conjecture 1 may (or may not) hold. We explicitly show these polynomials for $n = 3 .. 10$ along with their roots (using Mathematica) .

$$7 + 9x + 3x^2 - x^3$$

5.05456, 1.02728 ± 0.574095 i

$$15 + 28x + 18x^2 + 4x^3 - x^4$$

7.12129, -1.06065 ± 0.99065 i, -1.0

$$31 + 75x + 70x^2 + 30x^3 + 5x^4 - x^5$$

9.19269, -1.09666 ± 1.35988 i, -0.999688 ± 0.324918 i

$$63 + 186x + 225x^2 + 140x^3 + 45x^4 + 6x^5 - x^6$$

11.2664, -1.13492 ± 1.70789 i, -0.998264 ± 0.577325 i, -1.0

$$127 + 441x + 651x^2 + 525x^3 + 245x^4 + 63x^5 + 7x^6 - x^7$$

13.3413, -1.17494 ± 2.04444 i, -0.995724 ± 0.797366 i, -1.0 ± 0.228243 i

$$255 + 1016x + 1764x^2 + 1736x^3 + 1050x^4 + 392x^5 + 84x^6 + 8x^7 - x^8$$

15.4171, -1.21631 ± 2.37409 i, -0.992201 ± 0.999726 i, -1.00003 ± 0.414214 i, -1.0

$$511 + 2295x + 4572x^2 + 5292x^3 + 3906x^4 + 1890x^5 + 588x^6 + 108x^7 + 9x^8 - x^9$$

17.4934, -1.25872 ± 2.69929 i, -0.98782 ± 1.19122 i, -1.00014 ± 0.57735 i, -1.0 ± 0.176327 i

$$1023 + 5110x + 11475x^2 + 15240x^3 + 13230x^4 + 7812x^5 + 3150x^6 + 840x^7 + 135x^8 + 10x^9 - x^{10}$$

19.5700, -1.3020 ± 3.0214 i, -0.9827 ± 1.3755 i, 1.0004 ± 0.7265 i, -0.9999996 ± 0.3249 i, -1.0

It is interesting to study the behavior of roots and how they spread as the exponent increases. For the eighth order polynomial, one pair of the roots appear to be $-1 \pm i(\sqrt{2} - 1)$. One also have to look into the other polynomials of the form $(x + a)^n - (x + b)^n - x^n$ to get the total picture.

Now we derive the asymptotic values for $\alpha = r / n$ where r is the real (and only) root of $G(x, l, m, n) = (x + l)^n - (x + m)^n - x^n = 0$. We have:

$$\begin{aligned} (r + l)^n - (r + m)^n - r^n &= 0 \\ \Rightarrow (1 + l / r)^n - (1 + m / r)^n - 1 &= 0 \\ \Rightarrow (1 + l / (n\alpha))^n - (1 + m / (n\alpha))^n - 1 &= 0 \end{aligned}$$

As $n \rightarrow \infty$, we have $e^{l/\alpha} - e^{m/\alpha} - 1 = 0$ and setting $\beta = e^{1/\alpha}$, we get:

$$\beta^l - \beta^m - 1 = 0$$

Solving for β , one can get $\alpha = l / \ln \beta$. For $l = 2$ and $m = 1$, we get $\alpha = 1 / \ln \phi$ as noted before.

It is interesting that this approach leads to the following assertion:

Assertion 2: Fermat's Last Theorem cannot be true as the exponent n approaches infinity as it will violate the transcendental of e .

When r is an integer, we note that α is rational (an algebraic integer) and if Fermat's Last Theorem is true as the exponent n approaches infinity, we have:

$$e^{l/\alpha} - e^{m/\alpha} - 1 = 0$$

implying that $e^{1/\alpha}$ satisfies the algebraic equation:

$$x^l - x^m - 1 = 0$$

since " e^a if a is algebraic and nonzero (by the [Lindemann-Weierstrass theorem](#)), and in particular, e itself is transcendental" (see wikipedia.org under transcendental numbers), the assertion follows.

From this we have the following corollary.

Corollary: Fermat's Last Theorem can have only finite number of solutions in exponent n . If it does not, then the transcendental of e is violated.

Of course we have the Wiles' proof that this is zero. Note Gerd Faltings proved in 1983 that for every $n > 2$ there are at most a finite number of coprime integers x, y, z with $x^n + y^n = z^n$. See the following page at:

http://www-groups.dcs.stand.ac.uk/~history/HistTopics/Fermat's_last_theorem.html

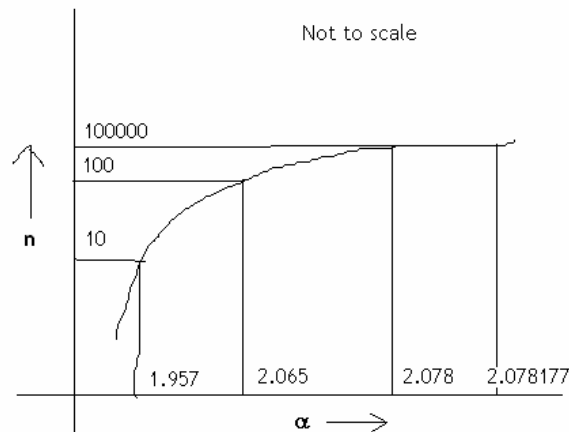
From Fermat's Last Theorem we can assert that $G(x, a, b, n) = (x + a)^n - (x + b)^n - x^n$ does not have zeros for $n > 3$ and hence irreducible over integers. From Conjecture1, we can extend that $G(x, a, b, n) - 3^n + 2^n + 1$ is also irreducible for $n > 5$ (disregarding the trivial solution $x = 1$ for $a = 2$ and $b = 1$). Such irreducible polynomials may have applications in encryption and encoding.

It is an interesting exercise to prove the irreducibility of $G(x, a, b, n)$ for $n = 3, 4$ and 5 by looking at the explicit solutions of the cubic, quartic and quintic equations. Also:

$$G(x, a, b, p) = -x^p + p(a - b)x^{p-1} + C(p, 2)(a^2 - b^2)x^{p-2} + \dots + (a^p - b^p)$$

and it is tempting to apply Eisenstein irreducibility criterion to prove that integer roots do not exist. We could not find interesting cases where p divides $(a^p - b^p)$ and p^2 does not divide $(a^p - b^p)$ for the criterion to hold.

Plotting n against $\alpha = r/n$ (Table 3) is reminiscent of an exponential function but it appears to be an extremely difficult task to derive this functionality. An explicit



derivation or characterization of this function (piece-wise or in entirety) may enable to prove why it always assumes irrational values for integer α thus proving the FLT. It is tempting to conjecture (or dream) that this function is composed of exponentials and rational values for α will contradict the transcendentalty of e thus giving a more rational and concise basis for the validity of the FLT.